Math 60 <u>9.9 The Complex Number System</u> (2 days)

Objectives:
1) Evaluate the square root of negative real numbers
2) Add or subtract complex numbers
3) Understand the difference between an imaginary number and a complex number.
4) Multiplying complex numbers
   - distribute
   - FOIL
   - $i^2 = -1$

We said before that $\sqrt{\text{neg}\#}$ = not a real number
Now we want to be more specific.

We will define $\sqrt{-1} = i$

i is always lower case
i is called the <u>imaginary unit</u>
i is the most basic form of a number that is not real.

Simplify.

① $\sqrt{-25}$ — notice radicand is a negative number

$= \sqrt{25 \cdot (-1)}$ — separate negative 1 from the positive number

$= \sqrt{25} \cdot \sqrt{-1}$ — use product property to separate

$= \boxed{5i}$ — simplify the square root, replace $\sqrt{-1}$ by i

② $\sqrt{-2}$

$= \sqrt{2 \cdot (-1)}$

$= \sqrt{2} \cdot \sqrt{-1}$

$= \sqrt{2} \cdot i$

$= \boxed{i\sqrt{2}}$

We usually write i last – except when a square root remains. Then we write i before the radical

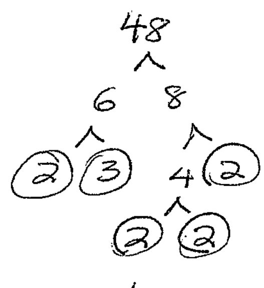
CAUTION
The i is outside the $\sqrt{\ }$
Do NOT WRITE $\sqrt{i}$

③ $\sqrt{-48}$

$= \sqrt{16 \cdot 3 \cdot (-1)}$ — separate −1 and perfect square factor

$= \sqrt{16} \cdot \sqrt{3} \cdot \sqrt{-1}$

$= 4\sqrt{3}\,i = \boxed{4i\sqrt{3}}$



$48 = 2^4 \cdot 3$
$= 4^2 \cdot 3$
$= 16 \cdot 3$

Math 60 9.9 – 1st

When a number is i multiplied by any number (rational or irrational), but no other terms, we call this number a <u>purely imaginary number</u>.

ex: $5i$, $i\sqrt{2}$, $-6i$, $4i\sqrt{3}$ are all purely imaginary numbers.

We can add a real number to a purely imaginary number

ex. $2+5i$, $1+i\sqrt{2}$, $3-6i$, $-4+4i\sqrt{3}$

These numbers have a <u>real part</u> and an <u>imaginary part</u>.

Simplify

(4) $3-\sqrt{-16}$

$= 3-\sqrt{16\cdot(-1)}$

$= 3-\sqrt{16}\cdot\sqrt{-1}$

$= \boxed{3-4i}$

We did nothing with the real part 3 except re-copy it.

(5) $5+\sqrt{-12}$

$= 5+\sqrt{4\cdot3\cdot(-1)}$

$= 5+\sqrt{4}\cdot\sqrt{3}\cdot\sqrt{-1}$

$= 5+2\sqrt{3}\,i$

$= \boxed{5+2i\sqrt{3}}$

(6) $\dfrac{15-\sqrt{-75}}{5}$

$= \dfrac{15-\sqrt{25\cdot(-1)\cdot3}}{5}$

simplify $\sqrt{-75}$ as before

$= \dfrac{15-\sqrt{25}\cdot\sqrt{-1}\cdot\sqrt{3}}{5}$

con't ⇒

Math 60 9.9 – 1st

$$= \frac{15 - 5i\sqrt{3}}{5}$$

divide each term, the real part 15 and the imaginary part $5i\sqrt{3}$, by 5

$$= \frac{15}{5} - \frac{5i\sqrt{3}}{5}$$

$$= \boxed{3 - i\sqrt{3}}$$

When we begin some problems, we may not know if the answer will be
- entirely a real number
- purely an imaginary number

or
- a real part and an imaginary part.

So we say that any number that can be written as $a + bi$, which is called <u>standard form</u>, where a is a real part and bi is an imaginary part is called a <u>complex number</u>,

EVEN IF $a = 0$
OR $b = 0$.

Write in the form $a + bi$:

(7) $2i = \boxed{0 + 2i}$ $a = 0$, $b = 2$

(8) $\sqrt{3} = \boxed{\sqrt{3} + 0i}$ $a = \sqrt{3}$, $b = 0$

(9) $6 = \boxed{6 + 0i}$ $a = 6$, $b = 0$

(10) $\frac{1}{3} = \boxed{\frac{1}{3} + 0i}$ $a = \frac{1}{3}$, $b = 0$

So <u>the set of complex numbers</u> includes all the real numbers and all the purely imaginary numbers as well as the numbers which have both real and imaginary parts.

This means that the set of complex numbers is the biggest set (has more numbers in it) we have discussed. In fact, <u>every</u> set we have discussed is contained in the set of complex numbers!

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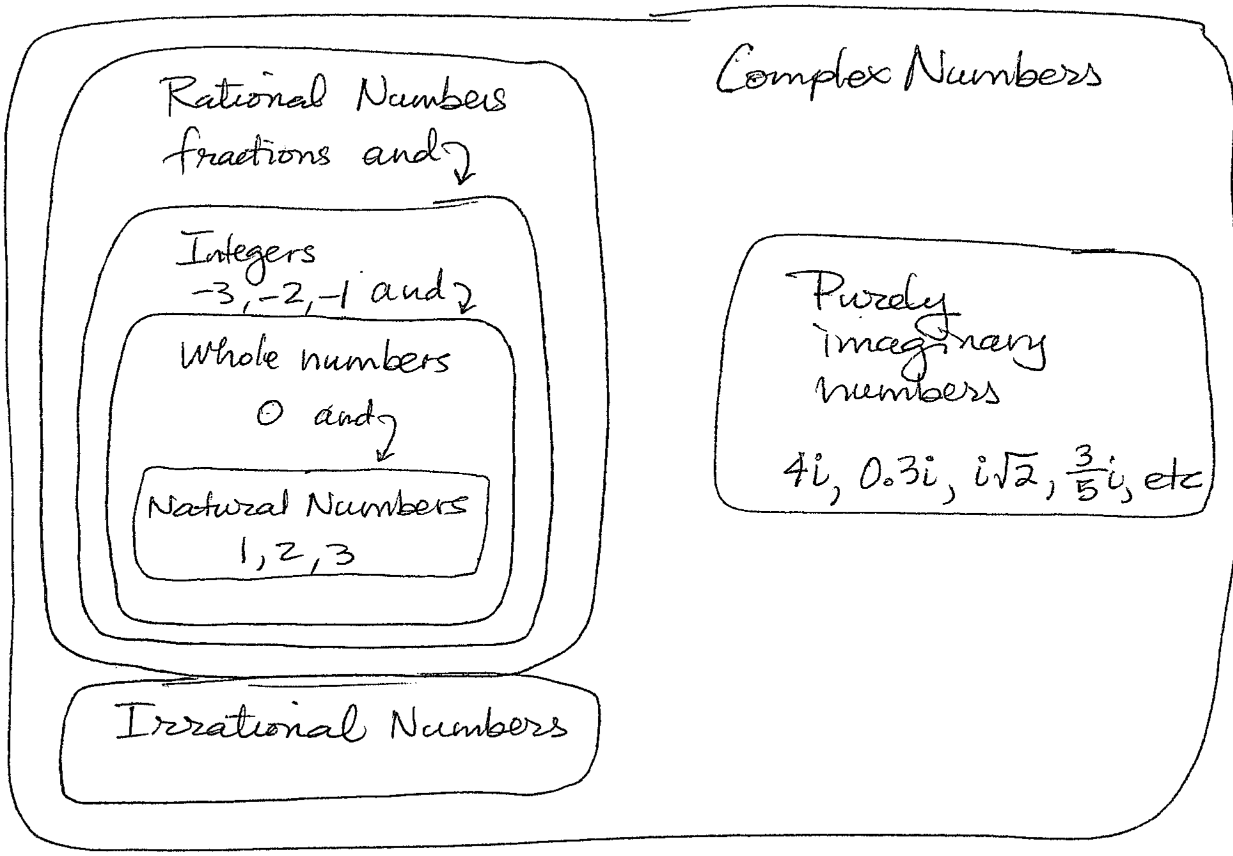

Recall:

Rational $\cup$ Irrational = Real

$\Uparrow$

union, join together

When we add or subtract complex numbers, we will add/subtract real parts to real parts and purely imaginary parts to purely imaginary parts.

{This is effectively the same as combining like terms}

Simplify

(11) $(2+3i)+(-6+7i)$

$= 2+3i+-6+7i$

$= 2-6+3i+7i$

$= \boxed{-4+10i}$

The parentheses are organizational only — they alert you that the first term is real and the second is imaginary.

Math 60 9.9 – 1st

(12) $\left(5+\sqrt{-36}\right)+\left(2-\sqrt{-49}\right)$

$= 5+\sqrt{36\cdot(-1)} + 2 - \sqrt{49\cdot(-1)}$ simplify radicals first

$= 5+6i+2-7i$ combine like parts

$= 5+2+6i-7i$

$= \boxed{7-i}$

(13) $(-1+5i)-(8+3i)$ just as we distribute the negative when subtracting polynomials, we do it here, too.

$= -1+5i-8-3i$

$= -1-8+5i-3i$

$= \boxed{-9+2i}$

(14) $\left(3+\sqrt{-16}\right)-\left(-2-\sqrt{-100}\right)$

$= \left(3+\sqrt{16\cdot(-1)}\right)-\left(-2-\sqrt{100\cdot(-1)}\right)$ simplify

$= (3+4i)-(-2-10i)$ dist neg

$= 3+4i+2+10i$ combine

$= \boxed{5+14i}$

If $i=\sqrt{-1}$
Then $i^2=\left(\sqrt{-1}\right)^2=-1$

CAUTION: Never write a final answer using i^2. Simplify $i^2=-1$.

(15) $2i(5-3i)$

This is a multiply.
We distribute $2i$ to both terms

$= 2i\cdot 5 - 2i\cdot 3i$

$= 2\cdot 5\cdot i - 2\cdot 3\cdot i\cdot i$ Use the commutative property of multiplication to change the order

$= 10i - 6\cdot i^2$

$= 10i - 6(-1)$

$= 10i+6 = \boxed{6+10i}$

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(16) $(5-2i)(-1+3i)$

This is a multiply.
Two terms × Two terms
means FOIL.

$$= \underset{F}{5(-1)} + \underset{O}{5\cdot 3i} - \underset{I}{2i(-1)} - \underset{L}{(2i)(3i)}$$

$$= -5 + 15i + 2i - 6i^2$$

$$= -5 + 17i - 6(-1)$$

$$= -5 + 6 + 17i$$

$$= \boxed{1 + 17i}$$

Note: $i = \sqrt{-1}$ is imaginary
but $i^2 = (-1)$ is real.

(17) $\sqrt{-49} \cdot \sqrt{-4}$

$$= \sqrt{49\cdot(-1)} \cdot \sqrt{4(-1)}$$

$$= (7i)\cdot(4i)$$

$$= 28i^2$$

$$= 28(-1)$$

$$= \boxed{-28}$$

Very important:
$\sqrt{a}\cdot\sqrt{b} = \sqrt{ab}$ ONLY WORKS
when $\sqrt{a}$ and $\sqrt{b}$ are real
numbers
$\sqrt{a}\cdot\sqrt{b} \neq \sqrt{ab}$ when $\sqrt{a}$ and $\sqrt{b}$
are both imaginary.

THIS MEANS:
⇒ Always simplify $\sqrt{neg}$ 1st
then multiply i by i

(18) $(3+\sqrt{-25})(1-\sqrt{-9})$

$$= (3+\sqrt{25(-1)})(1-\sqrt{9(-1)})$$

$$= (3+5i)(1-9i)$$

$$= 3 - 27i + 5i - 45i^2$$

$$= 3 - 22i - 45(-1)$$

$$= 3 - 22i + 45$$

$$= \boxed{48-22i}$$